EA - 2.2

1/ If $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$. Find X and Y.

$\rightarrow$ $$x+y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} \quad \text{—— (1)}$$

$$x-y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} \quad \text{—— (2)}$$

Adding eqn — (1) and (2)

$$x+\cancel{y}+x-\cancel{y}=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$\text{or, } 2x=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}$$

$$\text{or, } 2x=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\text{or } x=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

Putting the value in eqn — (1), we get,

$$\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\text{or, } y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

Ans

2/ $A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}$ Find $A+B$ and $A-B$.

$\rightarrow$ $A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix}$ ——— (1)

$B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}$ ——— (2)

Adding eqn — (1) and (2) we get,

$A+B = \begin{bmatrix} 2+0 & 3+5 & -5+1 \\ 1-2 & 2+7 & -1+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 8 & -4 \\ -1 & 9 & 2 \end{bmatrix}$

Substracting eqn — (1) and (2) we get,

$A-B = \begin{bmatrix} 2-0 & 3-5 & -5-1 \\ 1+2 & 2-7 & -1-3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -6 \\ 3 & -5 & -4 \end{bmatrix}$ Ans

3/ If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$. Find x

$\rightarrow$ $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$

or, $2x(x-2) - (-3)(3x) = 3$

or, $2x^2 - 4x + 9x = 3$

or, $2x^2 + 5x - 3 = 0$

or, $2x^2 + (6-1)x - 3 = 0$

or, $2x^2 + 6x - x - 3 = 0$

or, $2x(x+3) - 1(x+3) = 0$

or, $(x+3)(2x-1) = 0$

if,
$x+3=0$
or, $x=-3$

else,
$2x-1=0$
or, $x=\frac{1}{2}$

The value of x is
$\therefore x = \frac{1}{2}, -3$

Ans

4/ Let, $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. Find possible values of x and y if x, y are natural numbers.

→ $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

a, $3 - xy = 3 - 8$

a, $3 - xy = -5$

a $3 + 5 = xy$

a $xy = 8$

if, $x = 1$, $y = 8$
else, $x = 2$, $y = 4$
else, $x = 4$, $y = 2$
else, $x = 8$, $y = 1$

These are the possible values of x & y.

5/ Find the distance between the points $(2, 3, 1)$ and $(-1, 2, -3)$ using a vector Method.

→ The Formula for the magnitude of a vector $v = (x, y, z)$ is given

$$|v| = \sqrt{x^2 + y^2 + z^2}$$

Given, two pints $A(2, 3, 1)$ and $B(-1, 2, -3)$, the Vector $\vec{AB}$ is given by,

$$\vec{AB} = B - A = (-1-2, 2-3, -3-1)$$
$$= (-3, -1, -4)$$

The magnitude of this vector is

$$|\vec{AB}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-3)^2 + (-1)^2 + (-4)^2} = \sqrt{26}$$

Ans//

6/ Find $\vec{a} \cdot \vec{b}$ (1) $\vec{a} = 2i + 2j - k$ and $\vec{b} = 6i - 3j + 2k$

→ Given,

$\vec{a} = 2i + 2j - k$ and $\vec{b} = 6i - 3j + 2k$

We know that,

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\theta) \quad \text{--- (1)}$$

Now,

$$|\vec{a}| = \sqrt{(2)^2 + (2)^2 + (-1)^2} = 3$$

$$|\vec{b}| = \sqrt{(6)^2 + (-3)^2 + (2)^2} = 7$$

$$\therefore \vec{a} \cdot \vec{b} = (2 \times 6) + \{2 \times (-3)\} + \{(-1) \times 2\}$$

$$= 12 - 6 - 2$$

$$= 4$$

From eq^n - (1)

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

or, $\cos\theta = \dfrac{4}{3 \times 7}$

or, $\theta = \cos^{-1} \dfrac{4}{3 \times 7}$

$\therefore \theta \simeq 80°$

7/ An automobile having a mass of 2000 kg deflects its suspension spring 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible.

→ The formula for the natural frequency of a spring-mass system –

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

[where,
k = spring const, N/m
f = frequency, Hz
m = mass of the automobile, kg]

The spring constant K can be calculated using,

$K = F/x$ [where, F = Force applied to the string, N

Applied Force,

$\therefore F = mg$ x = deflection of the spring, m]

$= 2000 \times 9.81$ N

$F = 19{,}620$ N

$\therefore K = \dfrac{19{,}620}{0.02} = 981000$ N/m

$\therefore$ spring mass system frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{981000}{2000}}$$

$$= \frac{1}{2\times 3.14} \times 22.1$$

$$f \simeq 3.525 Hz.$$

8/ The natural frequency of a spring-mass system is found to be 2 Hz. When an additional mass of 1 kg is added to the original mass m, the natural frequency is reduced to 1Hz. Find the spring constant K and the mass m.

→ the natural frequency of a spring-mass system -

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\text{or, } 2 = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\text{or, } 4 = \frac{1}{4\pi^2}\times\frac{K}{m}$$

$$\text{or, } K = 16\pi^2 m \quad \text{———} (1)$$

When an additional mass of 1 kg is added, the total mass become m+1 kg. The natural frequency is reduced to 1 Hz.

$\therefore \quad 1 = \frac{1}{2\pi}\sqrt{\frac{k}{m+1}}$

Substituting the value of $k = 16\pi^2 m$ in above equation, we get -

$$1 = \frac{1}{2\pi}\sqrt{\frac{16\pi^2 m}{m+1}}$$

$$\text{or, } 1 = \frac{1}{\cancel{4\pi^2}} \times \frac{\cancel{16}^{4}\cancel{\pi^2} m}{m+1}$$

$$\text{or, } 1 = \frac{4m}{m+1}$$

$$\text{or, } m+1 = 4m$$

$$\text{or, } 3m = 1$$

$$\text{or, } m = \frac{1}{3}$$

Put the value of m in eqn - (1) we get,

$$K = 16\pi^2 \times \frac{1}{3}$$

$$\therefore K = 52.58 \text{ N/m} \quad \underline{\text{Ans}}$$

Q/ A spring-mass system has a natural period of 0.21 sec. What will be the new period if the spring constant is (a) increased by 50 percent and (b) decreased by 50 percent?

→ The formula for the period of a spring-mass-system, $T = 2\pi\sqrt{\frac{m}{k}}$

$$\text{or, } T_0 = 2\pi\sqrt{\frac{m}{k_0}} \quad [T_0 = 0.21 \text{ sec given}]$$

$$\text{or } 0.21 = 2\pi\sqrt{\frac{m}{k_0}}$$

$$\text{or, } 0.0441 = 4\pi^2 \times \frac{m}{k_0}$$

$$0.441 = \frac{4\pi^2 m}{K_0}$$

$$\text{or, } K_0 = \frac{4\pi^2 m}{0.441}$$

For case (a), when the spring constant is increased by 50%, the new spring constant K_1 is:

$$K_1 = K_0 + 0.5K_0 = 1.5K_0$$

For case (b), when the spring constant is decreased by 50%, the new spring constant K_2 is:

$$K_2 = K_0 - 0.5K_0 = 0.5K_0$$

Now, $T_1 = 2\pi\sqrt{\frac{m}{1.5K_0}} = 2\times3.14\times\sqrt{\frac{m}{4\pi^2 m\times1.5/0.441}}$

$$= 6.28\times\sqrt{\frac{\cancel{m}\times0.441}{4\pi^2\cancel{m}\times1.5}}$$

$$= 6.28\times0.086$$

$$T_1 = 0.54 \text{ sec}$$

Again, $T_2 = 2\pi\sqrt{\frac{m}{0.5K_0}}$

$$= 6.28\times\sqrt{\frac{\cancel{m}\times0.441}{4\pi^2\times\cancel{m}\times0.5}}$$

$$= 6.28\times0.149$$

$$T_2 = 0.93 \text{ sec} \quad \text{Ans}$$