Page No.:
Date:

Q If $x+y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x-y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Find x and y

→

let $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$y = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$x+y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

Add corresponding entries of the matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$a+e = 7$
$b+f = 0$
$c+g = 2$
$d+h = 5$

again

$x-y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

subtract the corresponding entries of the matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$a - e = 3$
$b - f = 0$
$c - g = 0$
$d - h = 3$
eliminate equations that contain e and f

$a + c = 7$
$b + d = 5$
$c = 2$
$d = 3$
solve eq^n for a
$a = 7 - 2$
$a = 5$
solve eq^n for b
$b = 5 - 3$
$b = 2$

Similarly solving
The required matrices are

$$x = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} = 15 - 4 = 11$$

$$y = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 4 - 0 = 4$$

Q $A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}$

Find $A+B$ and $A-B$

$\rightarrow$

$$A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}$$

Find $A+B$
Add the corresponding elements of the matrices

$$\begin{bmatrix} 2+0 & 3+5 & -5+1 \\ 1+(-2) & 2+7 & -1+3 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2 & 8 & -4 \\ -1 & 9 & 2 \end{bmatrix}$$

$A-B$ to find
Subtract the corresponding elements of the matrices

$$\begin{bmatrix} 2-0 & 3-5 & -5-1 \\ 1-(-2) & 2-7 & -1-3 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 2 & -2 & -6 \\ 3 & -5 & -4 \end{bmatrix}$$

Page No.: Date: YOUVA

Q If $\begin{bmatrix} x-2 & -3 \\ 3x & 2x \end{bmatrix} = 3$ Find x

$\rightarrow \begin{bmatrix} x-2 & -3 \\ 3x & 2x \end{bmatrix} = 3$

$\therefore \cancel{2x^2 - x + 9}$

$\therefore 2x(x-2) - (-3)3x = 3$

$\therefore 2x^2 + 5x - 3 = 0$

$\therefore 2x^2 + 6x - x - 3 = 0$

$(x+3)(2x-1) = 0$

$\therefore x = -3, \frac{1}{2}$

Q If $\begin{bmatrix} 3 & y \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$. Find possible values of x and y ~~w.r.t.~~ if x, y are natural no.

$\rightarrow$

$\begin{bmatrix} 3 & y \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$3 - xy = 3 \cancel{\times} 8$

$3 - xy = 3 - 8$

Page No.: Date: YOUVA

$3 - xx - 3 + 8 = 0$

$\therefore 5 - xy = 0$

It is in the form of

$ax + by = c$ where a, b, c are integers.

$\therefore$ Since x and y are natural number, their product $x \cdot y$ must be natural number

$\therefore$ Only solution to eq^n is

$x = 5, y = 1$

Q5 Find the distance between the points $(2, 3, 1)$ and $(-1, 2, -3)$ using vector method

$\rightarrow$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$

$= -\hat{i} + 2\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} - \hat{k}$

$= -3\hat{i} - \hat{j} + 2\hat{k}$

$\therefore$

Distance between the points

$= AB = \sqrt{(-3)^2 + (-1)^2 (+2)^2}$

$= \sqrt{9 + 1 + 4} = \sqrt{14}$ units.

Q Find $\bar{a}.\bar{b}$

$\bar{a} = 2i + 2j - k$ and $\bar{b} = 6\bar{i} - 3j + 2k$

→

$\bar{a} = 2i + 2j - k$ $\quad \bar{b} = 6i - 3j + 2k$

substitute

$a_1 = 2,\ a_2 = 2,\ a_3 = -1$

$b_1 = 6,\ b_2 = -3\ \ b_3 = 2$

into formula of dot product

$\bar{a}\cdot\bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\bar{a}\cdot\bar{b} = 2\times6 + 2\times(-3) + (-1)\times2$

$\bar{a}\cdot\bar{b} = 12 - 6 - 2$

Subtract the number

$\boxed{\bar{a}\cdot\bar{b} = 4}$

Page No.: Date: YOUVA

Q A automobile having a mass of 2000 kg deflects its suspension springs 0.02 m under static condition. Determine the natural frequency of automobile in the vertical direction by assuming damping to be negligible.

→ mass = 2000 kg

$\delta L = 0.02\,m$

under static condition

According to Hooke's Law,

$$\boxed{F = kx}$$

x = displacement

F = Force exerted by the spring

Weight of vehicle

$$\boxed{W = mg}$$

$W = F$

$\therefore mg = kx$

$\therefore k = \dfrac{mg}{x}$

$\therefore k = \dfrac{2000 \times 9.81}{0.02} = 981000\ N/m$

Spring constant $k = 981000\ N/m$

Natural frequency of spring mass system when damping is neglected

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{981000}{2000}} = 3.52\ Hz$$

The natural frequency of the automobile in the vertical direction is approximately 3.52 Hz

Q The natural frequency of spring mass system is found to be 2 Hz when a additional mass ~~is~~ of 1 kg is added to the original mass m, the natural frequency reduced to 1 Hz, Find the spring constant k and the mass m.

→ For the spring constant k and mass m in simple Harmonic motion,

$$\boxed{\frac{k}{m} = \omega^2}$$

ω = angular frequency (rad/sec)

$\omega = 2\pi f$

where

f = frequency (Hz)

$\therefore \frac{k}{m} = (2\pi f)^2$ — (1)

Substituting $m = M$ and $f = 2Hz$

$\frac{k}{M} = (2\pi \times 2)^2$ — (ii)

Substituting $m = M+1$ and $f = 1 Hz$

$\frac{k}{M+1} = (2\pi \times 1)^2$ — (iii)

Divide eq^n (2) by (3)

$4 = (M+1)/M$

$4M = M+1$

$3M = 1$

$M = \frac{1}{3} kg$

$\frac{K}{M} = (2\pi \times 2)^2$

substituting $\frac{1}{3}$ for M

$K = \frac{1}{3} \times 16\pi^2$

$K = 52.6$ N/m

original mass $= \frac{1}{3}$ kg

Spring constant is 52.6 N/m

Q A spring-mass system has a natural period of 0.21 sec. What will be the new period if the spring constant is
a) Increased by 50%.
b) decreased by 50%.

→ Initial period
$T_0 = 0.21$ sec.

Spring constant
- Increased by 50%.
$K_1 = 1.5K$
- Decreased by 50%.
$K_2 = 0.5K$

Page No.: Date:

New spring constant and the mass to initial - spring constant ratio into period formular

a) For increased spring constant

$$\boxed{T_1 = 2\pi\sqrt{\frac{m}{k_1}}}$$

$$= 2\pi\sqrt{\frac{m\cancel{k}}{k \cdot 1.5}}$$

$$= 2\pi\sqrt{\frac{T_0^2}{\frac{(2\pi)^2}{1.5}}}$$

$T_1 = 0.171\,Sec$

$$T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$= 2\pi\sqrt{\frac{m}{k \times 0.5}}$$

$$= 2\pi\sqrt{\frac{T_0^2}{\frac{(2\pi)^2}{0.5}}}$$

$T_2 = 0.297\,sec$

Scanned with CamScanner