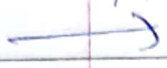


Q If $x+y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x-y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Find x and y



let $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$y = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$x+y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

Add corresponding entries of the matrices

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$a+e = 7$

$b+f = 0$

$c+g = 2$

$d+h = 5$

again

$x-y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

subtract the corresponding entries of the matrices

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\begin{aligned} a - e &= 3 \\ b - f &= 0 \\ c - g &= 0 \\ d - h &= 3 \end{aligned}$$

eliminate equations that contain e and f

$$\begin{aligned} a + c &= 7 \\ b + d &= 5 \\ c &= 2 \\ d &= 3 \end{aligned}$$

solve eqⁿ for a

$$\begin{aligned} a &= 7 - 2 \\ a &= 5 \end{aligned}$$

solve eqⁿ for b

$$\begin{aligned} b &= 5 - 3 \\ b &= 2 \end{aligned}$$

Similarly solving

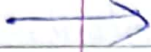
The required matrices are

$$x = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \Rightarrow 15 - 4 = 11$$

$$y = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow 4 - 0 = 4$$

Q $A = \begin{bmatrix} 3 & 2 & 3 & -5 \\ 1 & 2 & -1 & \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 & \end{bmatrix}$

Find $A+B$ and $A-B$



$$A = \begin{bmatrix} 3 & 2 & 3 & -5 \\ 1 & 2 & -1 & \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 & \end{bmatrix}$$

Find $A+B$

Add the corresponding elements of the matrices

$$\begin{bmatrix} 3+0 & 2+5 & 3+1 & -5+1 \\ 1+(-2) & 2+7 & -1+3 & \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 7 & 4 & -4 \\ -1 & 9 & 2 & \end{bmatrix}$$

$A-B$ to find

Subtract the corresponding elements of the matrices

$$\begin{bmatrix} 3-0 & 2-5 & 3-1 & -5-1 \\ 1-(-2) & 2-7 & -1-3 & \end{bmatrix}$$

$$A-B = \begin{bmatrix} 3 & -3 & 2 & -6 \\ 3 & -5 & -4 & \end{bmatrix}$$

Q If $\begin{bmatrix} x-2 & -3 \\ 3x & 2x \end{bmatrix} = 3$ Find x

$\rightarrow \begin{bmatrix} x-2 & -3 \\ 3x & 2x \end{bmatrix} = 3$

$\therefore 2x^2 - x + 9$

$\therefore 2x(x-2) - (-3)3x = 3$

$\therefore 2x^2 + 5x - 3 = 0$

$\therefore 2x^2 + 6x - x - 3 = 0$

$(x+3)(2x-1) = 0$

$\therefore x = -3, \frac{1}{2}$

Q If $\begin{bmatrix} 3 & y \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$. Find

possible values of x and y
 what if x, y are natural no.

\rightarrow

$\begin{bmatrix} 3 & y \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$3 - xy = 3 - 8$

$3 - xy = 3 - 8$

$$3 - 2x - 3 + 8 = 0$$

$$\therefore 5 - 2x = 0$$

It is in the form of

$ax + by = c$ where a, b, c are integers.

\therefore Since x and y are Natural numbers, their product xy must be Natural Number.

\therefore Only solution to eqn is $x=5, y=1$

Q Find the distance between the points $(2, 3, 1)$ and $(-1, 2, -3)$ using Vector method.

$$\rightarrow \vec{AB} = \vec{OB} - \vec{OA}$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$$

$$= -\hat{i} + 2\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} - \hat{k}$$

$$= -3\hat{i} - \hat{j} + 2\hat{k}$$

\therefore Distance between the points $= AB = \sqrt{(-3)^2 + (-1)^2 + (2)^2}$.

$$= \sqrt{9 + 1 + 4} = \sqrt{14} \text{ units.}$$

Q Find $\bar{a} \cdot \bar{b}$

$\bar{a} = 2i + 2j + k$ and $\bar{b} = 6i - 3j + 2k$

$$\rightarrow \bar{a} = 2i + 2j + k \quad \bar{b} = 6i - 3j + 2k$$

Substitute

$$a_1 = 2, a_2 = 2, a_3 = 1$$

$$b_1 = 6, b_2 = -3, b_3 = 2$$

into formula of dot product

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\bar{a} \cdot \bar{b} = 2 \times 6 + 2 \times (-3) + 1 \times 2$$

$$\bar{a} \cdot \bar{b} = 12 - 6 + 2$$

Subtract the number

$$\boxed{\bar{a} \cdot \bar{b} = 4}$$

Q A automobile having a mass of 2000 kg deflects its suspension springs 0.02 m under static condition. Determine the natural frequency of automobile in the vertical direction by assuming damping to be negligible.

→ mass = 2000 kg

$\Delta L = 0.02 \text{ m}$

under static condition

According to Hooke's Law,

$F = kx$

$x =$ displacement

$F =$ force exerted by the spring

Weight of vehicle

$\therefore W = mg$
 $W = F$

$\therefore mg = kx$

$\therefore k = \frac{mg}{x}$

$\therefore k = \frac{2000 \times 9.81}{0.02} = 981000 \text{ N/m}$

Spring constant $k = 981000 \text{ N/m}$

Natural frequency of spring mass system
 when damping is neglected

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{981000}{2000}} = 3.52 \text{ Hz}$$

The natural frequency of the automobile
 in the vertical direction is approximately
 3.52 Hz

Q The natural frequency of spring
 mass system is found to be 2 Hz
 When a additional mass is of
 1 kg is added to the original
 mass m , the natural frequency
 reduced to 1 Hz, find the spring
 constant k and the mass m .

→ For the spring constant k and
 mass m in simple harmonic
 motion,

$$\sqrt{\frac{k}{m}} = \omega^2$$

$$\omega = \text{angular frequency (rad/sec)}$$

$$\omega = 2\pi f$$

where

f = frequency (Hz)

$$\therefore \frac{k}{m} = (2\pi f)^2 \quad (1)$$

Substituting $m = M$ and $f = 2 \text{ Hz}$

$$\frac{k}{M} = (2\pi \times 2)^2 \quad (11)$$

Substituting $m = M \text{ H}$ and $f = 1 \text{ Hz}$

$$\frac{k}{M \text{ H}} = (2\pi \times 1)^2 \quad (11)$$

Divide eqⁿ (2) by (3)

$$4 = (M \text{ H}) / M$$

$$4M = M \text{ H}$$

$$3M = 1$$

$$M = \frac{1}{3} \text{ kg}$$

$$K = M(\omega)^2$$

substituting $\frac{1}{3}$ for M

$$K = \frac{1}{3} \times 1660$$

$$K = 52.6 \text{ N/m}$$

original mass = $\frac{1}{3} \text{ kg}$

Spring constant is 52.6 N/m

Q2 A spring-mass system has a natural period of 0.2 sec. What will be the new period if the spring constant is

- a) Increased by 50%.
- b) decreased by 50%.

→ Initial period

$$T_0 = 0.2 \text{ sec.}$$

Spring constant

- Increased by 50%.

$$k_1 = 1.5k$$

- Decreased by 50%.

$$k_2 = 0.5k$$

New Spring constant and the mass
to initial - Spring constant ratio into
period formula.

a) For increased spring constant

$$T_1 = 2\pi \sqrt{\frac{m}{k_1}}$$

$$= 2\pi \sqrt{\frac{m \cdot k}{k \cdot 1.5}}$$

$$= 2\pi \sqrt{\frac{T_0^2}{(2\pi)^2 \cdot 1.5}}$$

$$T_1 = 0.171 \text{ Sec}$$

$$T_2 = 2\pi \sqrt{\frac{m}{k_2}}$$

$$= 2\pi \sqrt{\frac{m}{k \cdot 0.5}}$$

$$= 2\pi \sqrt{\frac{T_0^2}{(2\pi)^2 \cdot 0.5}}$$

$$T_2 = 0.297 \text{ Sec}$$